

# High- $T_c$ Cuprate Superconductivity in a Nutshell

Hyekyung Won,<sup>1</sup> Stephan Haas,<sup>2</sup> David Parker\*,<sup>2</sup> and Kazumi Maki<sup>2</sup>

<sup>1</sup>*Department of Physics, Hallym University, Chuncheon 200-702, South Korea*

<sup>2</sup>*Department of Physics and Astronomy,*

*University of Southern California, Los Angeles, CA 90089-0484 USA*

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## Abstract

Since the discovery of high- $T_c$  cuprate superconductivity in 1986 many new experimental techniques and theoretical concepts have been developed. In particular it was shown that the BCS theory of d-wave superconductivity describes semi-quantitatively the high- $T_c$  superconductivity. Furthermore, it was demonstrated that Volovik's approach is extremely useful for finding the quasiparticle properties in the vortex state. Here we survey these developments and forecast future directions.

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\* Corresponding author: e-mail: davidspa@usc.edu, Phone: 213-740-1104

## 1. Introduction

In 1986 the epoch-making discovery of superconductivity in ceramic  $\text{La}_{2-x}\text{Ba}_x\text{CuO}_4$  with a transition temperature  $T_c = 35$  K by Bednorz and Müller [1] took the scientific community by surprise. The subsequent enthusiasm as well as the confusion in the theoretical community are well documented by Charles Enz [2].

In 1987 P.W. Anderson [3] published his “dogmas”. He pointed out that (a) all the action takes place in the Cu-O<sub>2</sub> plane common to all high- $T_c$  cuprates; and (b) the high- $T_c$  superconductivity has to be understood as arising in the middle of a Mott insulator at zero doping. He proposed a two-dimensional (2D) 1-band Hubbard model as the simplest model to describe the high- $T_c$  superconductivity. As a possible ground-state wavefunction, he proposed

$$\Phi = \prod_i (1 - d_i) |BCS\rangle, \quad (1)$$

where  $|BCS\rangle$  is the BCS wave function for s-wave superconductors [4],  $d_i \equiv n_{i\uparrow}n_{i\downarrow}$  and  $\Pi_i(1 - d_i)$  is the Gutzwiller projector, which annihilates all doubly occupied sites. We shall come back to Bob Laughlin’s observation on Eq.(1)[5].

Based on perturbative and numerical analysis of the 2D 1-band Hubbard model a possible d-wave superconductivity in high- $T_c$  cuprates was predicted by a few groups [6, 7, 8]. Meanwhile high-quality single crystals of YBCO, LSCO and thin-film Bi-2212 became available around 1992. D-wave superconductivity in high- $T_c$  cuprates was established finally around 1993-4 through ARPES [9] and Josephson interferometry [10, 11], among many other experiments.

## 2. BCS Theory of d-wave superconductors with impurities

Exploring d-wave superconductivity within the BCS framework [12, 13, 14, 15], we have shown that this theory describes quantitatively the observed quasiparticle density of states [16] and superfluid density [17] when Zn is substituted for Cu in the Cu-O<sub>2</sub> plane.

In 1993 Patrick Lee [18] discovered a remarkable phenomenon, i.e. the universal heat conduction. The quasi-particle spectrum in a d-wave superconductor is given by

$$E_k = \sqrt{v^2(k_{\parallel} - k_F)^2 + \Delta^2 \cos^2(2\phi)} \quad (2)$$

$$\simeq \sqrt{v^2(k_{\parallel} - k_F)^2 + v_2^2 k_{\perp}^2} \quad (3)$$

with  $v_2/v = \Delta/E_F$  and  $k_{\parallel}$  and  $k_{\perp}$  the components of  $\mathbf{k}$  parallel and perpendicular to the

nodal directions, respectively. Here the second equation is valid in the vicinity of the Dirac cones.

Then the thermal conductivity in the limit  $T \rightarrow 0$  and  $\Gamma \rightarrow 0$ ,  $\kappa_{00}$  is expressed as

$$\kappa_{00}/T = \frac{k_B^2 v}{3\hbar v_2} n \quad (4)$$

where  $n$  is the hole or electron density and  $E_F$  is the Fermi energy. Alternatively Eq. 4 can be rewritten as

$$\kappa_{00}/\kappa_n = \frac{2\Gamma}{\pi\Delta}, \quad (5)$$

with  $\Gamma$  the quasiparticle scattering rate in the normal state, and  $\kappa_n$  the thermal conductivity in the normal state. May Chiao et al [19, 20] then deduced  $\Delta/E_F = \frac{1}{10}$  and  $\frac{1}{14}$  for optimally doped Bi-2212 and YBCO respectively. Later the thermal conductivity measurement was extended to LSCO and Tl-2210 [21]. These ratios  $\Delta/E_F$  imply many things:

a) The pairing in high- $T_c$  cuprates is well described by the d-wave BCS theory. It is far away from the Bose-Einstein condensation limit.

b) According to the Ginzburg criterion, the fluctuation effects are of order  $\sim \Delta/E_F$ , i.e. they can be at most 10 percent. This appears to exclude the large phase fluctuation and stripe phase discussed in Refs. [22, 23].

c) For  $\Delta/E_F = 1/10$  there are hundreds of quasiparticle bound states around the core of a single vortex in d-wave superconductivity [24, 25]. Indeed the radial ( $r$ ) dependence of the quasiparticle density of states is very similar to the one obtained for s-wave superconductivity [26]. Here  $r$  is the distance from the center of the vortex. In earlier works [27, 28, 29, 30] it was claimed that there would be no bound states around a single vortex in d-wave superconductivity. However, in these works it was assumed that  $\Delta \simeq E_F$ . It is clear that this unrealistic assumption knocks off most of the bound states in this analysis. Unfortunately, these faulty works misled Hussey in his otherwise excellent review [31] on experiments in high- $T_c$  cuprate superconductors.

In summary, quasiparticles in d-wave superconductors behave as Landau-BCS quasiparticles. Also the quasiparticles of the normal state are part of a Landau Fermi liquid, although their properties are rather unorthodox.

### 3. Semiclassical Approximation

As discovered by Volovik [32], the quasiparticle spectrum of the vortex state in nodal superconductors is calculable within the semiclassical approximation. Later, Volovik's work

was extended for a planar magnetic field [33] and for thermal conductivity [34]. Unfortunately, however, Vohkter et al [33] have used an artificial and unrealistic Fermi surface, while Kübert et al [34] have introduced an unphysical spatial average. These problems were clarified and corrected in Refs. [35, 36, 37].

As is well known the quasiparticle energy in the presence of superflow is given [38] by

$$E_k \rightarrow E_k - \mathbf{v} \cdot \mathbf{q} \quad (6)$$

Here  $\mathbf{v}$  and  $2\mathbf{q}$  are the quasiparticle velocity and the pair momentum due to the superflow. Also  $\mathbf{v} \cdot \mathbf{q}$  is known as the Doppler shift (DS). Then the quasiparticle density of states on the Fermi surface  $N(\mathbf{H})$  is given by

$$N(\mathbf{H})/N(0) = G(\mathbf{H}) = \langle |\mathbf{v} \cdot \mathbf{q}| \rangle / \Delta \quad (7)$$

where  $\langle \dots \rangle$  means the average over both the Fermi surface and the vortex lattice. Then in a magnetic field  $\mathbf{H} \parallel \mathbf{c}$  in d-wave superconductors, we obtain

$$G(\mathbf{H}) = \frac{2}{\pi^2} \frac{v\sqrt{eH}}{\Delta} \quad (8)$$

Here  $v$  ( $= 2.6 \times 10^7$  cm/sec) is the Fermi velocity within the conduction plane. Recent ARPES indicate  $v$  is universal and independent of systems (YBCO, LSCO, Bi-2212) and of doping [9].

In all earlier analysis an extra  $\pi^{-1}$  factor is missing. This comes from

$$\frac{1}{2\pi} \int_0^{2\pi} d\phi \delta(\cos(2\phi)) = \frac{1}{\pi} \quad (9)$$

Actually this factor resolves the long-standing discrepancy between theory and experiment [39, 40]. When the magnetic field is applied within the a-b plane, we find [35, 37]

$$G(\mathbf{H}) \simeq \frac{2}{\pi^2} \frac{\tilde{v}\sqrt{eH}}{\Delta} (0.955 + 0.0285 \cos(4\phi)) \quad (10)$$

where  $\tilde{v} = \sqrt{v_c v}$  with  $v_c$  the Fermi velocity parallel to the c axis and  $\phi$  is the angle the magnetic field makes from the a axis. Since the low-temperature specific heat and the spin susceptibility are given by [41]

$$C_s/\gamma_N T = G(\mathbf{H}), \quad \chi/\chi_N = G(\mathbf{H}) \quad (11)$$

$G(\mathbf{H})$  should be readily accessible. However, the  $\phi$ -angle dependence of  $C_S$  in high- $T_c$  cuprate superconductors has not been seen yet.

On the other hand, a few thermal conductivity data of optimally doped YBCO have been reported which exhibit clear fourfold symmetry [42, 43, 44]. For  $T < (\Gamma\Delta)^{1/2} \ll \tilde{v}\sqrt{eH}$  we obtain [35]

$$\frac{\kappa_{xx}}{\kappa_n} = \frac{\kappa_{yy}}{\kappa_n} \simeq \frac{2\tilde{v}^2(eH)}{\pi^4\Delta^2}(0.955 + 0.0285\cos(4\phi))^2 \quad (12)$$

Moreover, for  $T \gg \tilde{v}\sqrt{eH}$ , the sign of the fourfold term becomes negative[37]. This is observed experimentally in [42, 43, 44].

More recently the quasiparticle density of states in the vortex state of a variety of other nodal superconductors has been analyzed[45, 46]. Using thermal conductivity measurements, Izawa et al have succeeded in identifying  $\Delta(\mathbf{k})$  of  $\text{Sr}_2\text{RuO}_4$  [47],  $\text{CeCoIn}_5$ [48],  $\kappa\text{-(ET)}_2\text{Cu(NCS)}_2$  [49],  $\text{YNi}_2\text{B}_2\text{C}$  [50] and  $\text{PrOs}_4\text{Sb}_{12}$  [51]. These  $\Delta(\mathbf{k})$ 's are shown in Fig. 1. In analogy to Eq.(12), the magnetothermal conductivity data in high quality single crystals at low temperature ( $T \ll \Delta$ ) provides unique access to the nodal structure of  $\Delta(\mathbf{k})$ .

#### 4. D-wave Density Waves and Gossamer Superconductivity

In Fig. 2 we sketch the phase diagram of the hole doped high- $T_c$  cuprate superconductors. These materials occupy the region  $0.05 < x < 0.25$ , where  $x$  is the hole concentration. In the insulating side ( $0 \leq x < 0.03$ ) the antiferromagnetic phase is realized. The phase space below  $T^*$  is called the pseudogap region whose nature has been hotly debated. One may suspect that under the cover of the pseudogap phase many different phases are hidden. It was proposed [52, 53, 54] that the pseudogap phase is an unconventional density wave (UDW). UDW is a density wave in which the quasiparticle energy gap has nodes. Therefore, the transition from the normal state to UDW is a metal-metal transition, although the quasiparticle density decreases rapidly in UDW. Furthermore, the local charge density or the spin density in UDW is hard to observe since  $\langle \Delta(\mathbf{k}) \rangle = 0$ , where  $\langle \dots \rangle$  denotes the average over the Fermi surface. Therefore UDW is often called a condensate with a “hidden order parameter”[53]. For more about UDW we suggest the reader to study Ref. [55]. ARPES data in the pseudogap region indicates clearly the d-wave nature of  $\Delta(\mathbf{k})$  [56] (i.e.  $\Delta(\mathbf{k}) \sim \cos(2\phi)$ ).

Although the evidence for d-wave DW or dDW is still elusive, the giant negative Nernst effect observed in the underdoped region of LSCO, YBCO and Bi-2212 by Wang et al[57, 58]

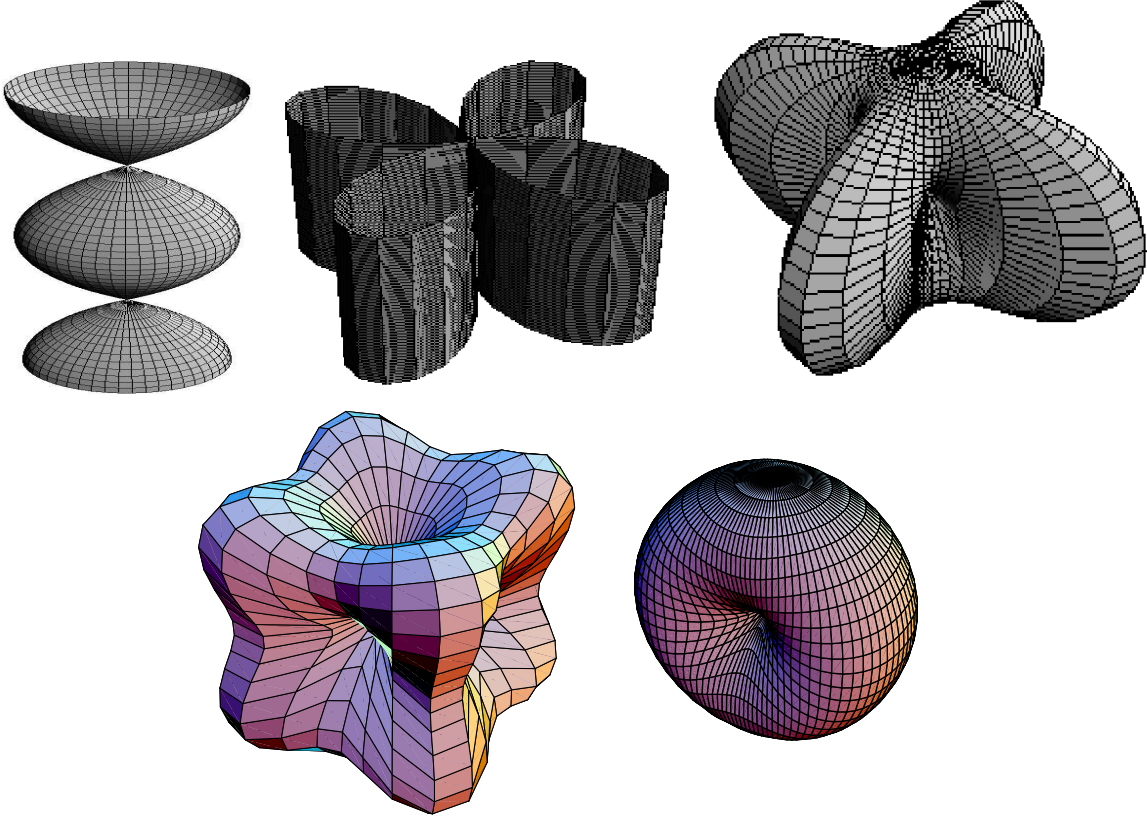


FIG. 1: From top left, 2D f-wave -  $\text{Sr}_2\text{RuO}_4$ ,  $d_{x^2-y^2}$ -wave -  $\text{CeCoIn}_5$  and  $\kappa$ -( $\text{ET}$ ) $_2\text{Cu}(\text{NCS})_2$ , s+g-wave -  $\text{YNi}_2\text{B}_2\text{C}$ , p+h-wave -  $\text{PrOs}_4\text{Sb}_{12}$  - A phase, p+h-wave -  $\text{PrOs}_4\text{Sb}_{12}$  - B phase.

indicates UDW. It was previously shown that the giant negative Nernst effect is the hallmark of UDW [59]. Also, UDW appears to describe the observed large Nernst effect in underdoped LSCO, YBCO and Bi-2212 very consistently [60]. In the phase diagram of high- $T_c$  cuprates (Fig. 2), it is very likely that 2 order parameters dDW and dSC (d-wave superconductivity) coexist in the limited region. This problem has been briefly discussed in Ref. [53]. In another paper, Laughlin has presented an intuitive interpretation of Eq.(1). The mathematical difficulty of Eq.(1) comes from the Gutzwiller operator, which has no inverse. If one replaces  $\prod_i (1 - d_i)$  by  $\prod_i e^{-\alpha d_i}$ , Eq.(1) can be understood as a state with competing order parameters. The fragile superconductivity in the Mott insulator is called “gossamer superconductivity” [5]. However, in d-wave superconductivity the Coulomb potential is not so devastating. Furthermore, in the region where the superconductivity arises the anti-ferromagnetic state has already disappeared. Instead, the dominant condensate is dDW. Therefore the competition between dDW and superconductivity as discussed in [53, 61] is

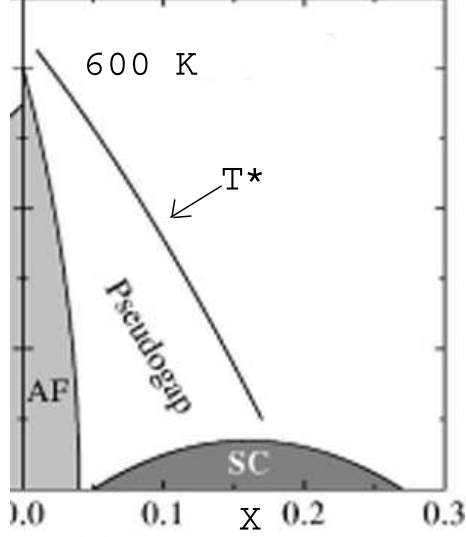


FIG. 2: Phase diagram of the hole-doped cuprates from Ref. [9]

more realistic. In the following we shall use the term “gossamer superconductivity” for d-wave superconductivity in the presence of dDW.

In a pure dDW phase the thermal conductivity exhibits universal heat conduction as long as imperfect nesting or the chemical potential is neglected [62]. This observation is unaffected in the presence of two competing order parameters as long as the imperfect nesting or the chemical potential is negligible compared to  $T$  or  $(\Gamma\Delta)^{1/2}$ . Then we recover Eq.(3)

$$\kappa_{00}/T = \frac{k_B^2 E_F n}{3\hbar\Delta} \quad (13)$$

where  $\Delta = \sqrt{\Delta_1^2 + \Delta_2^2}$  and  $\Delta_1$  and  $\Delta_2$  are the order parameters of dDW and d-wave superconductivity respectively. In the pseudogap region the thermal conductivity can measure  $\Delta \simeq \Delta_1$ . This is shown in Fig. 3 (Fig. 6 in [21]). Clearly  $\Delta_0 \simeq \Delta_{dDW}(0)$  is close to  $2.14 T^*$ . Indeed a very similar  $x$  dependence of  $\Delta$  observed by STM has been reported [63, 64]. Note that 2.14 is the weak-coupling theory value for both the d-wave superconductor [12] and d-wave density-wave [65].

## 5. Concluding Remarks

The 20th anniversary of the discovery of high- $T_c$  cuprate superconductivity is coming soon. In the meanwhile, we have learned a lot about the properties of both the normal state and the superconducting state in quasi-2D systems and many other systems. When the effects of disorder can be neglected, these quasiparticles behave as a Fermi liquid although

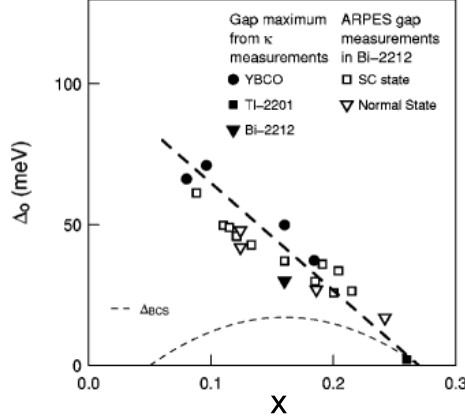


FIG. 3: Gap amplitude for various cuprates in the pseudogap phase

some details may be unorthodox. Landau's Fermi liquid is thus the most universal normal state in high- $T_c$  cuprate superconductors, heavy-fermion superconductors and organic conductors.

Furthermore, all the ground states that have been found belong to one of three groups: (a) nodal superconductors which can be spin singlet or spin triplet; (b) nodal density waves which can be charge density wave (CDW) or spin density wave (SDW), and (c) the coexistence of nodal superconductors and nodal density waves.

The high- $T_c$  cuprate superconductors contain all these ground states in their rich phase diagram. We are discovering many parallels between high- $T_c$  cuprate superconductivity and CeCoIn<sub>5</sub> [66] and  $\kappa$ -(ET)<sub>2</sub>Cu(N(CN)<sub>2</sub>)Br [67]. Unconventional density waves have been identified in  $\alpha$ -(ET)<sub>2</sub>KHg(SCN)<sub>4</sub> [68] and (TMTSF)<sub>2</sub>PF<sub>6</sub> [69] through angle dependent magnetoresistance measurements, and in CeCoIn<sub>5</sub> [66] and in the pseudogap phase of high- $T_c$  cuprates [60] through the giant Nernst effect.

In all nodal superconductors, the determination of their gap symmetry is the first crucial step. This allows us to construct the effective Hamiltonian to describe the plethora of new ground states.

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